IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 05, Issue 03 (March. 2015), ||V3|| PP 01-07

wavelet collocation method for solving integro-differential equation.

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Abstract: - Wavelet collocation method for numerical solution nth order Volterra integro diferential equations (VIDE) by expanding the unknown functions, as series in terms of chebyshev wavelets second kind with unknown coefficients. The aim of this paper is to state and prove the uniform convergence theorem and accuracy estimation for series above. Finally, some illustrative examples are given to demonstrate the validity and applicability of the proposed method.

Keywords: *chebyshev wavelets second kind; integro-differential equation; operational matrix of integrations; uniform convergence; accuracy estimation.*

I. INTRODUCTION

Basic wavelet theory is a natural topic. By name, wavelets date back only to the 1980s. on the boundary between mathematics and engineering, wavelet theory shows students that mathematics research is still thriving, with important applications in areas such as image compression and the numerical solution of differential equations [1], integral equation [2],and integro differential equations [3,8]. The author believes that the essentials of wavelet theory are sufficiently elementary to be taught successfully to advanced undergraduates [4].

Orthgonal functions and polynomials series have received considerable attention in dealing with various problems wavelets permit the accurate representation of a functions and operators. Special attention has been given to application of the Legendre wavelets [5], Harr wavelets [6] and Sine-Cosine wavelets [7].

The solution of integro-differential equations have a major role in the fields of science and ingineering when aphysical system is modeled under the differential sense, it finally gives a differential equation, an integral equation or an integro-differential equations mostly appear in the last equation [8].

In this paper the operational matrix of integration for Hermite wavelets is derived and used it for obtaining approximate solution of the following nth order VIDE.

$$u^{(n)}(x)=g(x)+\int_0^x k(x,t)u^{(s)}(t)dt \qquad (1)$$

where $n \geq s$ k(x,t) and g(x) are known functions, and u(x) is an unknown function.

II.SOME PROPERTIES OF SECOND CHEBYSHEV WAVELETS

Wavelets constitute a family of functions constructed from dilation and traslation of a single function $\omega(t)$ called the mother wavelet.When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets as [9].

$$\omega_{a,b}(t) = |a|^{\frac{-1}{2}}\omega\left(\frac{t-b}{a}\right), a, b \in R, \ a \neq 0.$$

The second chebyshev wavelets $\omega^2{}_{n,m}(t) = \omega(k,n,m,t)$ involve four arguments, $n = 1, \ldots, 2^{k-1}$, k is assumed any positive integer, m is the degree of the second chebyshev polynomials and t is the normalized time. They are defined on the interval $[0,1)$as

$$\omega^2{}_{n,m}(t) = \begin{cases} 2^{\frac{k}{2}} \quad \tilde{U}_m(2^k t - 2n + 1), \frac{n-1}{2^{k-1}} \leq t < \frac{n}{2^{k-1}} \\ 0 \qquad\qquad otherwise \end{cases} \qquad (2)$$

where $\tilde{U}_m(t) = \sqrt{\frac{2}{\pi}}U_m(t) \qquad m = 0,1, \ldots, M-1 \qquad (3)$

here $U_m(t)$ are the second chebyshev polynomialsof degree m with respect to the weight function $w(t) = \sqrt{1-t^2}$ on the interval [-1,1] and satisfy the following recursive formula

$$U_0(t) = 1, U_1(t) = 2t,$$
$$U_{m+1}(t) = 2tU_m(t) - U_{m-1}(t), \ m = 1,2, \ldots$$

The set of chebyshev wavelets are an orthonirmal set with respect to the weight function

$$w_n(t) = w_n(2^k t - 2n + 1) \ .$$

III.FUNCTION APPROXIMAT

A function $f(t)$ defined over [0, 1) may be expanded as

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} C_{nm}\, \omega_{nm}^2(t) \qquad (4)$$

$$where \qquad C_{nm} = (f(t), \omega_{nm}^2(t))$$

In which $(.,.)$ denoted the inner product in $L_{w_n}^2[0,1)$. If the infinite series equation (2.34) is truncated, then it can be written

$$f(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} C_{nm}\, \omega_{nm}^2(t) = C^T \omega^2(t) \qquad (5)$$

$C = \left[C_{10}, C_{11}, \dots, C_{1(M-1)}, C_{20}, \dots, C_{2(M-1)}, \dots, C_{2^{k-1}}, \dots, C_{2^{k-1}M-1}\right]^T$ $\omega^2(t) = \left[\omega_{10}^2(t), \omega_{11}^2(t), \dots, \omega_{1M-1}^2(t), \omega_{20}^2(t), \dots, \omega_{2^{k-1}M-1}^2(t), \dots \omega_{2^{k-1}0}^2(t), \dots \omega_{2^{k-1}M-1}^2(t)\right]^T$

Convergence Analysis for Chebyshev wavelets of Second kind.

A function $f(t)$ defined over $[0,1]$ may be expanded as:

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} f_{nm}\, \omega_{nm}^2(t) \qquad (6)$$

where

$$f_{nm} = \left(f(t), \omega_{nm}^2(t)\right) \qquad (7)$$

In eq.(5).$(.,.)$ denotes the inner product with weight function $w_n(t)$.
If the infinite series in eq.(4) is trancated then eq.(4) can be written as:

$$f(t) \simeq f_{2^{k-1}M-1}(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} f_{nm}\, \omega_{nm}^2 = F^T \omega_{nm}^2$$

where F and ω_{nm}^2 are $2^k M \times 1$ matrices given by

$$F = \left[f_{10}, f_{11}, \dots, f_{1M}, f_{20}, \dots, f_{2M-1}, \dots, f_{2^k 0}, \dots, f_{2^{k-1}M-1}\right]^T \qquad (8)$$

$$\omega^2(t) = \left[\omega_{10}^2(t), \omega_{11}^2(t), \dots, \omega_{1M-1}^2(t), \omega_{20}^2(t), \dots, \omega_{2M-1}^2(t), \omega_{2^{k-1}}^2(t), \dots, \omega_{2^{k-1}M-1}^2(t)\right]^T$$

Theorem(1): (Convergence Analysis theorm)

Assume that a function f(t) $\in L_{w^*}^2[0,1], w^* = \sqrt{1-t^2}$ with $|f''(t)| \le L$, can be expanded as infinite series of second kind chebyshev wavelets, then the series converges uniformly to f(t).

Proof: since $f_{nm} = (f(t), \omega_{nm}^2(t))$
then

$$f_{nm} = \int_0^1 f(t)\, \omega_{nm}^2(t) w_n(t) dt$$

$$= \int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} \frac{2^{\frac{k+1}{2}}}{\sqrt{\pi}} f(t) U_m(2^k t - 2n + 1) w(2^k t - 2n + 1) dt \qquad (9)$$

If we make use of the substitution $2^k t - 2n + 1 = \cos\theta$ in (5), yields

$$f_{nm} = \frac{\sqrt{2}}{2^{\frac{k}{2}}\sqrt{\pi}} \int_0^{\pi} f\left(\frac{\cos\theta + 2n - 1}{2^k}\right) \sin\theta \sin(m+1)\theta\, d\theta \qquad (10)$$

By using the integration by parts,
Then eq.(10) becomes

$$f_{nm} = \frac{1}{\sqrt{2}\, 2^{\frac{k}{2}}\sqrt{\pi}} \left[\left[f\left(\frac{\cos\theta + 2n - 1}{2^k}\right) \left(\frac{\sin m\theta}{m} - \frac{\sin(m+2)\theta}{m+2}\right) \right] \Bigg|_0^{\pi} + \frac{1}{\sqrt{2}\, 2^{\frac{3k}{2}}\sqrt{\pi}} \int_0^{\pi} f'\left(\frac{\cos\theta + 2n - 1}{2^k}\right) \sin\theta \left(\frac{\sin m\theta}{m} - \frac{\sin(m+2)\theta}{m+2}\right) \right] d\theta$$

$$f_{nm} = \frac{1}{m\sqrt{2}\, 2^{\frac{3k}{2}}\sqrt{\pi}} \int_0^{\pi} f'\left(\frac{\cos\theta + 2n - 1}{2^k}\right) \sin\theta \sin m\theta d\theta - \frac{1}{(m+2)\sqrt{2}\, 2^{\frac{3k}{2}}\sqrt{\pi}} \int_0^{\pi} f'\left(\frac{\cos\theta + 2n - 1}{2^k}\right) \sin\theta \sin(m+2)\theta d\theta \qquad (11)$$

After performing integration by parts again yields,

$$f_{nm} = \frac{1}{m2^{\frac{3}{2}}\, 2^{\frac{5k}{2}}\sqrt{\pi}} \int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m-2)\theta - \cos m\theta}{m-1} - \frac{\cos m\theta - \cos(m+2)\theta}{m+1}\right] d\theta$$
$$- \frac{1}{(m+2)2^{\frac{3}{2}}\, 2^{\frac{5k}{2}}\sqrt{\pi}} \int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m)\theta - \cos(m+2)\theta}{m+1} - \frac{\cos(m+2)\theta - \cos(m+4)\theta}{m+3}\right] d\theta \tag{12}$$

Consider:

$$\left|\int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m-2)\theta - \cos m\theta}{m-1} - \frac{\cos m\theta - \cos(m+2)\theta}{m+1}\right] d\theta\right|^2$$
$$= \left|\int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m-2)\theta - \cos m\theta}{m-1} - \frac{\cos m\theta - \cos(m+2)\theta}{m+1}\right] d\theta\right|^2$$
$$\leq \int_0^\pi \left|f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\right|^2 d\theta \times \int_0^\pi \left|\frac{(m-1)\cos(m+2)\theta - 2m\cos m\,\theta + (m+1)\cos(m-2)\theta}{(m-1)(m+1)}\right|^2 d\theta$$
$$< \pi L^2 \int_0^\pi \frac{(m-1)^2\cos^2(m+2)\theta + 4m^2\cos^2 m\theta + (m+1)^2\cos^2(m-1)}{(m-1)^2(m+1)^2} d\theta$$
$$= \frac{\pi L^2}{(m-1)^2(m+1)^2}\left[\frac{\pi}{2}(m-1)^2 + \frac{\pi}{2}4m^2 + \frac{\pi}{2}(m+1)^2\right]$$
$$= \frac{\pi^2 L^2}{(m-1)^2(m+1)^2}[3m^2+1]$$

Thus, we get

$$\left|\int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m-2)\theta - \cos m\theta}{m-1} - \frac{\cos m\theta - \cos(m+2)\theta}{m+1}\right] d\theta\right| < \frac{\pi L(3m^2+1)^{1/2}}{(m-1)(m+1)} \tag{13}$$

Similarly,

$$\left|\int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m)\theta - \cos(m+2)\theta}{m+1} - \frac{\cos(m+2)\theta - \cos(m+4)\theta}{m+3}\right] d\theta\right|^2$$
$$= \left|\int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right) \times \frac{(m+1)\cos(m+4)\theta - (m+1)\cos(m+2)\,\theta + (m+3)\cos(m+2)\theta + (m+3)\cos(m)\theta}{(m+1)(m+3)}\right|^2$$
$$\leq \int_0^\pi \left|f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\right|^2 d\theta \times \int_0^\pi \left|\frac{m\cos(m+3)\theta - (2m-2)\cos(m+1)\,\theta + (m+2\theta)\cos(m-1)\theta}{m(m+2)}\right|^2$$
$$< \pi L^2 \int_0^\pi \frac{(m+1)^2\cos^2(m+4)\theta + (2m+4)^2\cos^2(m+2)\theta + (m+3)^2\cos^2(m)\theta}{(m+1)^2(m+3)^2}$$
$$= \frac{\pi^2 L^2}{2(m+1)^2(m+3)^2}[6m^2+24m+26]$$
$$= \frac{\pi^2 L^2}{(m+1)^2(m+3)^2}[3m^2+12m+13]$$

Thus we get

$$\left|\int_0^\pi f"\left(\frac{\cos\theta + 2n - 1}{2^k}\right)\left[\frac{\cos(m)\theta - \cos(m+2)\theta}{m+1} - \frac{\cos(m+2)\theta - \cos(m+4)\theta}{m+3}\right]\right| d\theta < \frac{\pi L}{(m+1)(m+3)}(3m^2+12m+13)^{1/2} \tag{14}$$

Using eqs.(13)and(14), one can get

$$|f_{nm}| < 2^{-\frac{5k-3}{2}}\pi^{1/2}\left(\frac{\pi L(3m^2+1)^{1/2}}{m(m-1)(m+1)} - \frac{\pi L(3m^2+12m+13)^{1/2}}{(m+2)(m+1)(m+3)}\right)$$

$|f_{nm}| < \frac{2^{-\frac{5k}{2}}\pi^{1/2}}{2^{\frac{3}{2}}(m+1)}\left(\frac{2(m+1)}{(m-1)^2}\right) = \frac{2^{-\frac{5k}{2}}\pi^{\frac{1}{2}}L}{2^{\frac{1}{2}}(m-1)^2}$

Finally since $n \le 2^k - 1$,then

$$|f_{nm}| < \frac{\pi^{\frac{1}{2}}L(n+1)^{-\frac{5}{2}}}{\sqrt{2}(m-1)^2}$$

Accuracy Estimation of $\omega_{nm}^2(x)$:

If the function f(x) is expanded interms of fourth kind chebyshev wavelets,

$$f(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} f_{nm}\,\omega_{nm}^2(x) \tag{15}$$

It is not possible to perform computation an infinite number of terms, therfore we must truncate the series in (15). In place of (5), we take

$$f_M(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} f_{nm}\,\omega_{nm}^2(x) \tag{16}$$

so that

$$f(x) = f_M(x) + \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f_{nm}\,\omega_{nm}^2(x)$$

or
$$f(x) - f_M(x) = r(x) \tag{17}$$

where r(x) is the residual function

$$r(x) = \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f_{nm}\,\omega_{nm}^2(x) \tag{18}$$

we must select coefficients in eqs.(17) and (18) such that the norm of the residual function $\|r(x)\|$ is less than some convergence criterion $\in$,that is

$$\left(\int_0^1 (f(x) - f_M)^2 w_n(x)dx\right)^{\frac{1}{2}} < \in$$

for all M greater than some value M_0.

Theorem (2)

Let f(x) be a continuaus function defined on [0,1), and $|f''(x)| < L$, then we have the following accuracy estimation

$$c_{k,M} < \frac{\pi L}{2\sqrt{2}}\sum_{n-2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty}\frac{1}{(n+1)^5}\frac{1}{(m-1)^4} \tag{19}$$

where

$$c_{k,M} = \left(\int_0^1 (r(x))^2 w_n(x)\,dx\right)^{\frac{1}{2}}$$

<u>Proof</u>:-

Since $\boldsymbol{C_{km} = \left(\int_0^1 (r(x))^2 w_n(x)\,dx\right)^{\frac{1}{2}}}$

$$C_{km}^2 = \int_0^1 (r(x))^2 w_n(x)\boldsymbol{dx}$$

$=\int_0^1 \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f_{nm}^2\left(\omega^2 w_n(x)\right)dx \qquad = \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f_{nm}^2\int_0^1 (\omega_{nm}^2(x))^2 w_n(x)\,dx$

or

$$C_{km}^2 = \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty}\int_0^1 f_{nm}^2(\omega^2)^2 w_n(x)dx$$

$$C_{km}^2 = \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f^2\left(2^{\frac{k}{2}}\right)^2\int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} U_m(2^k x - 2n + 1)^2\sqrt{1-(2^k x - 2n + 1)^2}dx$$

Let $t = 2^k x - 2n + 1$,

$$C_{km}^2 = \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f^2\int_{-1}^{1} U_m^2(t)\sqrt{1-t^2}dx$$

we have,

$$\int_{-1}^{1} U_m^2(t)\sqrt{1-t^2}dx = \frac{\pi}{2}$$

then

$$C_{km}^2 = \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty} f^2\frac{\pi}{2}$$

$$C_{km}^2 < \sum_{n=2^{k-1}+1}^{\infty}\sum_{m=M}^{\infty}\frac{\pi^2 L^2(m+1)^{-5}}{8(m-1)^4}\ .$$

wavelet collocation method for VIDE with mth order:

In this section the introduced wavelets collocation will be applied to solve VIDE with mth order,

$$u_i^{(n)}(x) = g_i(x) + \int_0^x K_{i,j}(x,t)u_i^{(s)}(t)dt\,, n \geq s \qquad (20)$$

With the following conditions $u_i^s(0) = a_{is}$ $\quad i = 1,2,\ldots,l \quad s = 0,1,2,\ldots,n-1$

Afunction $u_i^n(x)$ which is defined on the interval $x \in [0,1]$ can be expanded into the second chebyshev wavelet series

$$u_i^n(x) = \sum_{i=1}^{M} c_i\omega_i(t) \qquad (21)$$

Where c_i are the wavelet coefficients.

Integrate eq.(21) m times,yields

$$u(x) = \sum_{i=0}^{M} c_i \int_0^x \ldots \int_0^x \omega_i(t)dt + \sum_{j=0}^{m-1} \frac{x^j}{j!} a_{m-j} \qquad (22)$$

Using the following formula

$$\int_0^x \ldots \int_0^x \omega_i(t)dt = \frac{1}{(n-1)!}\int_0^x (x-t)^{n-1}\omega_i(t)dt$$

therefore eq.(22) becomes

$$u(x) = \sum_{i=0}^{M} c_i \frac{1}{(n-1)!}\int_0^x (x-t)^{n-1}\omega_i(t)dt + \sum_{j=0}^{n-1}\frac{x^j}{j!}a_{n-j} \qquad (23)$$

Let $K_n(x,t) = \frac{(x-t)^{n-1}}{(n-1)!}$ and $L_i^n = \int_0^x K_n(x,t)h_i(t)dt$ i=0,1,…,M

This leads to

$$u(x) = \sum_{i=0}^{M} c_i L_i^n + \sum_{j=0}^{n-1}\frac{x^j}{j!}a_{n-j}$$

In similar way, we can get

$$u^{(s)}(x) = \sum_{i=0}^{M} c_i L_i^{n-s} + \sum_{j=0}^{n-s-1}\frac{x^j}{j!}a_{n-s-j} \qquad (24)$$

Substituting eqs (22) and (24) in (20), yield

$$\sum_{i=1}^{M} c_i\omega_i(t) = g_i(x) + \int_0^x K_{i,j}(x,t)\left[\sum_{i=0}^{M} c_i L_i^{n-s} + \sum_{j=0}^{n-s-1}\frac{x^j}{j!}a_{n-s-j}\right]dt \qquad (25)$$

or

$$\sum_{i=1}^{M} c_i\omega_i(t) - A_i(x) = g_i(x) + \sum_{j=0}^{n-s-1}\frac{a_{n-s-j}}{j!}B_j(x) \qquad (26)$$

where $A_i(x) = \int_0^x K_n(x,t)L_i^{n-s}(t)dt$ i=0,1,2,…,M

$$B_j(x) = \int_0^x K_n(x,t)t^j\,dt \quad \text{j=0,1,2,…,n-s-1} \qquad (27)$$

Next the interval $x \in [0,1]$ is devided in to l $\Delta x = \frac{1}{l}$ and introduce the collocation points $x_k = \frac{k-1}{l}$, k=1,2,…,l eq(21) is satisfied only at the collocation points we get asystem of linear equations

$$\sum_{i=1}^{M} c_i[\omega_i(x) - A_i(x)] = g_i(x) + \sum_{j=0}^{n-s-1}\frac{a_{n-s-j}}{j!}B_j(x) \qquad (28)$$

The matrix form of this system is C F=G+ $\sum_{j=0}^{n-s-1}\frac{a_{n-s-j}}{j!}B_j(x)$ where F=ω(x), G=g(x)

1.Design of the matrix A:-

When chebyshev wavelets second kind are integrated m times, the following integral must be evaluated.

$L_i^n = \int_0^x K_n(x,t)\omega_i(t)dt$, i=0, 1, 2, …, M

$$L_i^n(x) = \frac{(x-t)^n}{2^k(n-1)!}\begin{bmatrix} 1 & \frac{1}{2} & 0 & \ldots & 0 & \vdots & 2 & 0 & \ldots & 0 \\ \frac{-3}{4} & 0 & \frac{1}{4} & \ldots & 0 & \vdots & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{1}{6} & 0 & \ldots & 0 & \vdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \frac{1}{2(M-1)} & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{-1^{M-2}}{M} & 0 & 0 & \ldots & 0 & \vdots & 0 & 0 & \ldots & 0 \end{bmatrix} \qquad \frac{l-1}{2^k} \leq x < \frac{l}{2^k}$$

Therefore the matrix $A_i(x)$ can be constructed as follows

Since $A_i(x) = \int_0^x K_n(x,t)L_i^{n-s}(t)dt$ i=0,1,2,…,M

$$A_i(x) = \begin{bmatrix} \int_0^{x_0} K_n(x_0,t)L_i^{n-s}(t)dt & i=0 \\ \int_0^{x_n} K_n(x_i,t)L_i^{n-s}(t)dt & i>0 \end{bmatrix}$$

IV. WAVELETS METHOD FOR VIDE WITH NTH ORDER

For solving VIDE with mth order the matrix $L_i^n(x)$ in section above will be followed to get

$$\sum_{i=1}^{M} c_i[\omega_i(x_L - A_L)] = g(x_L) + \sum_{j=0}^{n-s-1} \frac{a_{n-s-j}}{j!} B_j(x_L) \qquad L \in [a,b]$$

But $A_i(x_L) = \int_0^{x_L} K_n(x_L,t)L_i^{n-s}(t)dt$ where i=0,…,M

$B_j(x_L) = \int_0^{x_L} K_n(x_L,t)t^{n-s}dt$

where $L_i^{n-s}(t)$ as in eq(17),(18)

that is $A_i(x_L) = A_L$, $F_i(x_L) = \omega_i(x_L) - A_i(x_L) = F_L$

Numerical Results:

In this section VIDE is considered and solved by the introduced method. parameters k and M are considered to be 1 and 3 respectively.

Example 1: Consider the following VIDE:

$$U''(x) = e^{2x} - \int_0^x e^{2(x-t)}U'(t)dt$$

Initial conditions $U(0) = 0, U'(0) = 0$.

The exact solution $U(x) = xe^x - e^x + 1$. Table 1 shows the numerical results for this example with k=2, M=3 with error $=10^{-3}$ and k=2, M=4, with error $=10^{-4}$ are compared with exact solution graphically in fig.

Table 1:some numerical results for example 1

x	Exact solution	Approximat solution k=2,M=3	Approximat solution k=2,M=4
0	0.00000000	0.00000001	0.00000001
0.2	0.02287779	0.02280000	0.02287000
0.4	0.10940518	0.10945544	0.10940544
0.6	0.27115248	0.25826756	0.27826756
0.8	0.55489181	0.54330957	0.55330957
1	1.00000000	0.99999995	0.99999998

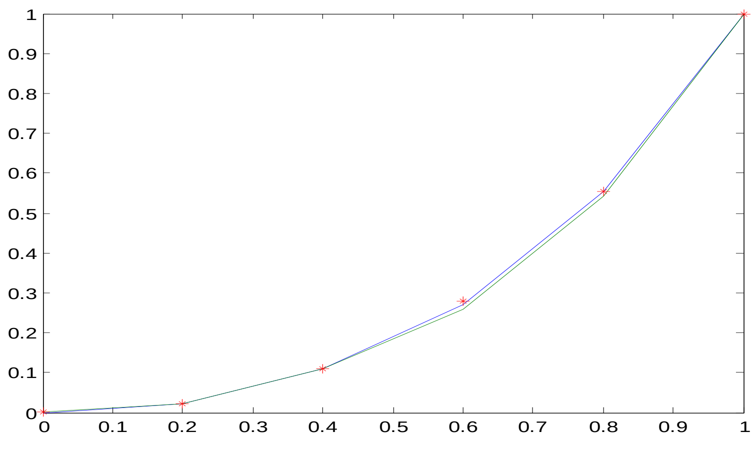

Fig 1:Approximate solution for example 1

Example 2: Consider the following VIDE :

$$U^{(5)}(x) = -2\sin x + 2\cos x - x + \int_0^x (x-t)U^{(3)}(t)dt$$

Initial conditions $U(0) = 1, U'(0) = 0, U''(0) = -1, U^3(0) = 0$.

The exact solution $U(x) = \cos x$. Table 2 shows the numerical results for this example with k=2, M=3 with error$=10^{-3}$ and k=2, M=4, with error $=10^{-4}$ are compared with exact solution graphically in fig, 2.

Table 2:some numerical results for example 2

x	Exact solution	Approximat solution k=2,M=3	Approximat solution k=2,M=4
0	1.00000000	0.99812235	0.99999875
0.2	0.98006658	0.98024711	0.98005541
0.4	0.92106099	0.92158990	0.92104326
0.6	0.82533561	0.82479820	0.82535367
0.8	0.69670671	0.69689632	0.69678976
1	0.54030231	0.54032879	0.54035879

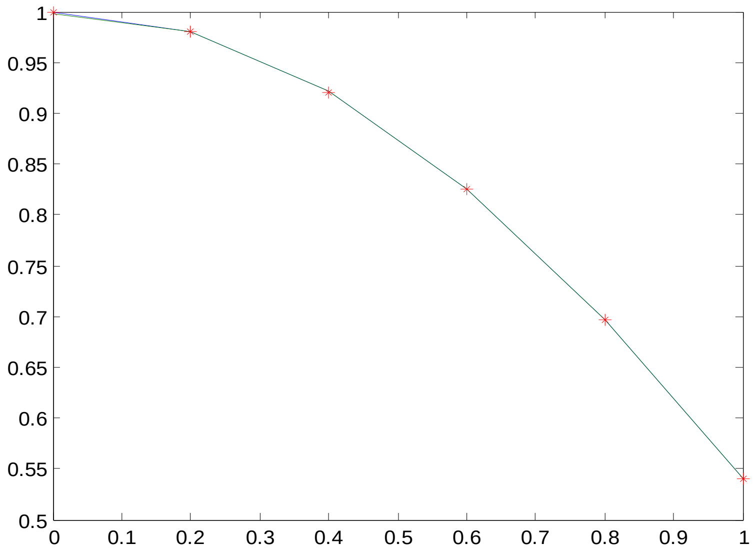

Fig 2:Approximate solution for example

V. CONCLUSION

This work proposes a powerful technique for solving VIDE second kind using wavelet in collocation method comparison of the approximate solutions and the exact solutions shows that the proposed method is more faster algorithms than ordinary ones. The convergence and accuracy estimation of this method was examined for several numerical examples.

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